A Formal Model of Reliable Sensor Perception

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Abstract. The safety of autonomously acting systems depends on a reliable assessment of the systems' context. We propose a framework to formalise and analyse both qualitative and quantitative measures of the context quality in terms of safety and precision. The measures are based on order-theoretic arguments by relating the ground truth (as given by the real environment) with the context information (as inferred by the sensors). We derive a hierarchy of qualitative notions that can serve as a high-level requirement description for the sensor and controller implementation of the system. The quantitative part of the framework then allows for an evaluation of a probabilistic variant of the sensor regarding its safety and precision. We in particular treat the analysis of sensor fusion based on evidence theory.

1 Introduction

Context-awareness is an important prerequisite for enabling pervasive computing applications and smart behaviour of systems. Any autonomously acting system needs to have reliable information about the relevant part of its environment in order to exhibit a safe and effective behaviour. Consider for example an autonomously driving vehicle. Here, an important context information is whether some obstacle is located within the planned driving trajectory. This context information is typically acquired by the use of sensors, which monitor the area in front of the vehicle. There exists a number of techniques and algorithms that infer relevant context information from the raw sensor data. In almost every scenario however, the context information is only an abstraction of the real environment of the system. For example, while the sensor may properly detect the presence or absence of an obstacle, the exact kind of obstacle (e.g. whether it is car or a lorry) can often not be detected.

Assessing the Quality of Context (QoC) is relevant for any kind of ubiquitous computing. In general, the quality is influenced by a number of parameters [9,1], like the timeliness, trust-worthiness, completeness and significance of information. Now we are particular interested in autonomous behaviour of safety-critical systems, i.e. systems whose malfunction may cause severe damage or even harm to persons. For these systems, QoC plays an important role [18,12,8] and a thorough investigation of the safety aspects of context becomes indispensable.

In our proposal, we derive two fundamental parameters for determining the quality of context for such kind of systems, namely *safety* and *precision*. When sensing a real environment and mapping it to its context representation, a safe context means that the information is not “wrong” with respect to the reality, while a precise context means that the information is not “coarser than necessary”. An unsafe context may obviously lead to hazards, and an imprecise context may prevent any meaningful behaviour of the system.

We will formalise both notions of safety and precision by the use of order-theory. To this end, we regard a context perception as an abstraction relation between two “worlds”, namely between the real environment and the systems' context. Properties of this relation can then be expressed in terms of two functions going back and forth between these two worlds. While the first function corresponds to the act of *sensing*, the second function reflects how the system *interprets* the context in terms of possible environment situations. Our claim is that in fact the relationship of *both* functions has to be considered in order to define a meaningful notion of safety and precision (Sect. 2). The results of this investigation can be seen as a formal representation of requirements on both the sensors (expressed in terms of the sensing function) and the control algorithms of the system (expressed in terms of the interpretation function).

However, as real-world sensors are typically not fully reliable, the adherence to the sensing function cannot be guaranteed in all cases. To this end, we also propose in Section 3 an extension towards a *quantitative* notion of safety and precision. We do so by relating a probabilistic variant of the sensing function to our formalisation of a (qualitative) safe and precise perception. In Section 4 we discuss related work, and Section 5 concludes.

2 Sensor Perception

As a running example, we consider a simple environment comprising three possible environment situations $\mathcal{E} = \{\mathit{car}, \mathit{person}, \mathit{nothing}\}$, namely the presence of a car, of a person and the absence of any obstacle. A context is represented by means of a set of exclusive context predicates [19]. For example, in order to represent a sensor that detects the presence of obstacles, we use the two predicates $\mathcal{C} = \{\mathit{occ}, \mathit{free}\}$, which allows us to represent four different contexts:

$$C_0 = \emptyset \qquad C_1 = \{\mathit{occ}\} \qquad C_2 = \{\mathit{free}\} \qquad C_3 = \{\mathit{occ}, \mathit{free}\}$$

C_0 represents the impossible context where no predicate is satisfied. C_1 and C_2 represent definite context information, stating that the area is occupied or free, respectively. C_3 represents an indefinite context information where both predicates are possibly true.

As already sketched above, our notion of context quality involves two mappings, namely a sensing ς from an environment situation to a context, and an interpretation ι of a context back to the environment. Then, a safe perception requires a safe interpretation of a sensing $\varsigma(e)$ of an environment situation e, that is, we have to state

How does $\iota(\varsigma(e))$ relate to e?

In general, one does not require an equality relation here, because that would imply that the sensing is exact and does not loose any information. Intuitively, to guarantee a safe but not necessarily exact perception, the combination $\iota(\varsigma(e))$ should represent "at least e, but possible more". In the running example, it would be meaningful to require

$$\iota(\varsigma(car)) = \{car, person\}$$

if $\varsigma(car)$ yields $\{occ\}$ and the fact that the sensor has detected the presence of an object should be interpreted as having some object, either a car or a person, in the sensed environment. We will formalise this intuition in the following section.

2.1 Classification of Sensor Perception

The powerset of a set S is defined as $\mathcal{P}(S) := \{S' \mid S' \subseteq S\}$. Powersets are a particular instance of a complete lattice [4], that is, a partially ordered set $(S, \leq)$ where any subset has both a least upper bound (lub) and a greatest lower bound (glb). Lattices exhibit a canonical notion of information order induced by the partial order: the relation $S_1 \leq S_2$ states that S_1 represents at least as precise information as S_2. Likewise, $S_1 \lneq S_2$ indicates strictly more precise information of S_1 wrt. S_2. For powersets, the order '$\leq$' is given by set inclusion '$\subseteq$' and the lub and glb are obtained by set union and intersection, respectively. Note that in our example, we in particular have $C_1 \subsetneq C_3$ and $C_2 \subsetneq C_3$.

We now define and characterise a perception based on the properties of its sensing and interpretation. We will first motivate these properties, then formally define them in Def. 2 and finally give examples for some properties in Fig. 1.

Definition 1 (Perception). *A tuple $(\mathcal{E}, \varsigma, \iota, \mathcal{C})$ where*

- *$\mathcal{E}$ is a finite set of environment situations,*
- *$\mathcal{C}$ is a finite set of exclusive context predicates,*
- *$\varsigma : \mathcal{P}(\mathcal{E}) \to \mathcal{P}(\mathcal{C})$ is a sensing function, and*
- *$\iota : \mathcal{P}(\mathcal{C}) \to \mathcal{P}(\mathcal{E})$ is an interpretation function,*

is called a perception. ◊

A first criterion is whether both functions preserve the ordering of information, that is, whether they are monotone functions. The second criterion addresses the combination of sensing and interpretation. Starting with an argument e of the sensing function, a suitable characterisation for being safe is that the interpretation of the obtained sensing yields e or some less precise information than e. However, we should not obtain information that is more precise or unrelated to e. The third criterion considers the combination of first applying the interpretation and then the sensing function. Again, we will demand a proper ordering of information, however in the opposite direction as for safety. As we will see below, this property allows us to characterise the (qualitative) precision of a perception. Moreover, we can strengthen the previous two criteria by requiring equality instead of orderedness.

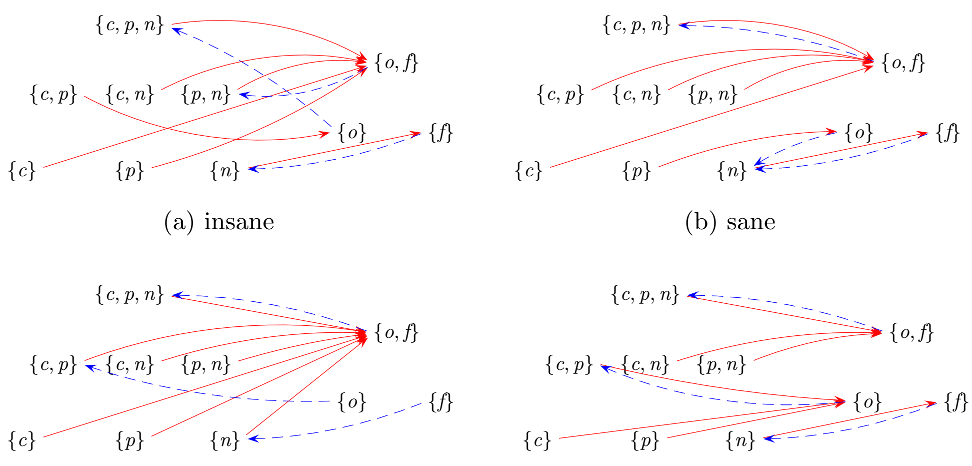
Definition 2 (Perception Classification). *A perception $(\mathcal{E}, \varsigma, \iota, \mathcal{C})$ is called*

- *sane if ς and ι are monotone functions, i.e.*

$$E_1 \subseteq E_2 \implies \varsigma(E_1) \subseteq \varsigma(E_2) \quad \text{and} \quad O_1 \subseteq O_2 \implies \iota(O_1) \subseteq \iota(O_2)$$

- *safe if it is sane and $\forall E \in \mathcal{P}(\mathcal{E}) : E \subseteq \iota(\varsigma(E))$,*
- *precise if it is safe and $\forall O \in \mathcal{P}(\mathcal{C}) : \varsigma(\iota(O)) \subseteq O$,*
- *concise if it is precise and $\forall O \in \mathcal{P}(\mathcal{C}) : \varsigma(\iota(O)) = O$,*
- *exact if it is concise and $\forall E \in \mathcal{P}(\mathcal{E}) : \iota(\varsigma(E)) = E$.* ♢

A number of perceptions for the running example are given in Figure 1. For space reasons, all elements are abbreviated by their first letter, and the relation between the empty sets, $\varsigma(\emptyset) = \emptyset$ and $\iota(\emptyset) = \emptyset$, are omitted.



(a) insane (b) sane (c) safe (d) concise

Fig. 1: Characterisation of perceptions by analysing the composition of sensing (solid arrows) and interpretation (dashed arrows) functions.

Perception 1a is not sane since neither ι nor ς are monotone. For example, we have $\iota(\{o, f\}) = \{c, p\}$ and $\iota(\{o\}) = \{c, p, n\}$, that is, ι derives more precise information from $\{o, f\}$ than from $\{o\}$. Perception 1b is sane but not safe. We have $\varsigma(\{p\}) = \{o\}$ and $\iota(\{o\}) = \{n\}$, hence $\{p\} \not\subseteq \iota(\varsigma(\{p\}))$. In fact, interpreting the sensing of a person as the empty situation should not be considered to be a safe perception. Perception 1c is safe as the sensing of every non-empty environment is interpreted to be the universal environment $\mathcal{E}$. This coarseness however leads to, e.g., $\iota(\{f\}) = \{n\}$ and $\varsigma(\{n\}) = \{o, f\}$, hence $\varsigma(\iota(\{f\})) \not\subseteq \{f\}$. Thus, the perception is not precise according to Def. 2. Perception 1d is concise, hence also precise. It matches with our intuition that both $\{c\}$ (car) and $\{p\}$ (person) are merged to $\{o\}$ (occ), and $\{n\}$ (nothing) to $\{f\}$ (free). The indefinite context occurs if a combination of n with c or p is sensed. Also, the interpretation yields the most precise value with respect to the given sensing function, e.g. $\iota(\{o\}) = \{c, p\}$. Note that precision is to be understood with respect to $\mathcal{C}$, that is, a precise perception preserves as much information as possible, given the set of context predicates. In our example, the kind of obstacle is dismissed as both situations c and p are sensed to the same context. In contrast, an exact perception has no loss of information at all, which is only possible if $|\mathcal{E}| = |\mathcal{C}|$.

The most relevant notions are that of *precise* and *concise* perceptions. A perception that is not precise does not effectively use its potential. Conciseness yields additional benefits by ensuring that ς becomes surjective (hence every possible context is actually used by the perception) and ι becomes injective (hence we have meaningful interpretations for all non-empty contexts, see below). To require an exact perception seems not to be realistic for real-world applications.

2.2 Relation to Galois Connections

From an order-theoretic perspective [4], the notion of a precise perception corresponds to the definition of a *Galois connection*, and a concise perception to that of a *Galois insertion*. A number of results from the theory of Galois connection immediately transfers to our characterisation of precise and concise perceptions. The proofs of these results can be found in [11]. The first proposition states that a precise perception is fully determined by either one of the sensing and interpretation function.

Proposition 1 (Determination). *Let $(\mathcal{E}, \varsigma, \iota, \mathcal{C})$ be a precise perception. Then ς uniquely determines ι by $\iota(O) = \bigcup\{E \mid \varsigma(E) \subseteq O\}$ and ι uniquely determines ς by $\varsigma(E) = \bigcap\{O \mid \iota(O) \subseteq E\}$ for $E \subseteq \mathcal{E}$ and $C \subseteq \mathcal{C}$.* $\diamondsuit$

Related to this observation, we can automatically obtain a precise perception via a basic sensor function that maps from $\mathcal{E}$ to $\mathcal{P}(\mathcal{C})$.

Proposition 2 (Basic sensor function). *Let $\varsigma_0 : \mathcal{E} \to \mathcal{P}(\mathcal{C})$ be a function. Then the perception $(\mathcal{E}, \varsigma, \iota, \mathcal{C})$ with*

$$\varsigma(E) := \bigcup_{e \in E} \varsigma_0(e)$$

$$\iota(C) := \{e \in \mathcal{E} \mid \varsigma_0(e) \subseteq C\}$$

for $E \subseteq \mathcal{E}$ and $C \subseteq \mathcal{C}$ is precise. $\diamondsuit$

This yields a methodology for constructing precise perceptions by first choosing the sets $\mathcal{E}$ and $\mathcal{C}$ according to the real-word scenario, then defining a basic sensor function $\varsigma_0 : \mathcal{E} \to \mathcal{P}(\mathcal{C})$ which describes for each environment situation a suitable context, and apply proposition 2. We illustrate this construction on the running example, by using a basic sensor function ς_0 with

$$\varsigma_0(c) = \{o\} \qquad \varsigma_0(p) = \{f\} \qquad \varsigma_0(n) = \{f\}$$

This yields a perception $(\mathcal{E}, \varsigma, \iota, \mathcal{C})$ with $\varsigma(\{e\}) = \varsigma_0(e)$ for $e \in \mathcal{E}$ and

$$\begin{array}{ccc} \varsigma(\{c,p\}) = \{o,f\} & \varsigma(\{c,p,n\}) = \{o,f\} & \iota(\{f\}) = \{p,n\} \\ \varsigma(\{c,n\}) = \{o,f\} & \varsigma(\emptyset) = \emptyset & \iota(\{o\}) = \{c\} \\ \varsigma(\{p,n\}) = \{f\} & \iota(\emptyset) = \emptyset & \iota(\{o,f\}) = \{c,p,n\} \end{array}$$

We obtain a rather unintuitive perception as e.g. both the environments '$\{p\}$' (person) and '$\{n\}$' (nothing) are sensed to the context '$\{f\}$' (free). As this context however is appropriately interpreted by ι, the perception as a whole is safe and precise. This again underlines the importance of considering both mappings to obtain a meaningful characterisation of context quality. Moreover, this example demonstrates that there is not a unique precise perception for a given pair of environment and context. Note that this construction also ensures that only the empty environment is sensed to the empty (impossible) context. Additionally, if every environment situation e is in the image of the basic sensor function, we obtain a concise perception in which ι becomes injective, and hence the same holds for the interpretation function (that is, $\iota(C) = \emptyset$ implies $C = \emptyset$).

Finally, the theory of Galois connections equips us with a notion of compositionality. This is of practical relevance whenever the context recognition cannot be performed in one sweep but requires multiple stages of sensing. In this case, the following result allows us to analyse each stage in isolation and still conclude properties for the whole perception chain.

Proposition 3 (Composition). *Let $(\mathcal{E}, \varsigma, \iota, \mathcal{C})$ and $(\mathcal{E}', \varsigma', \iota', \mathcal{C}')$ be precise perceptions with $\mathcal{C} = \mathcal{E}'$. Then $(\mathcal{E}, \varsigma' \circ \varsigma, \iota \circ \iota', \mathcal{C}')$ is a precise perception.* ♢

2.3 Discussion

The presented classification of perceptions allows for a qualitative assessment in terms of safety and precision. In particular, our framework can be seen as a formalisation of system requirements, separated into requirements on the sensor part (given by the sensing function) and requirements on the controller part (given by the interpretation function). These requirements express the desired mappings for each environment situation and context. Still, the sensing function can in particular map a singleton environment situation to a *set* of exclusive context predicates, thereby expressing an inherent uncertainty in observing this environment situation. We will see that this mapping affects the notion of quantitative safety and precision as defined below.

All in all, the characterisation of a precise or concise perception as a *reference* can help to exclude systematic errors in the design of the perception process. However, a perfect realisation of the sensing function based on hardware sensors and corresponding software algorithms is in general not possible. This is as real-words sensors comprise an inherent amount of inaccuracy and may be disturbed by a number of external influences. Also, software algorithms typically employ heuristics for deriving context information out of raw data and thus cannot be fully reliable in general. In order to account for this fact, we propose in the following a quantitative extension of our approach in order to be able to measure the overall quality (both in terms of safety and precision) of a probabilistic variant of the sensing function with respect to a given perception.

3 Uncertainty

As motivated above, real-world sensors are in general not fully reliable and hence cannot be faithfully formalised as a function that maps each environment situation to one dedicated context. Rather, a sensor corresponds to a probabilistic mapping to different contexts. For example, while a sensor will most of the time (e.g. with probability 0.99) properly detect a car, in some rare cases (e.g. with probability 0.01) it fails and erroneously yields e.g. the value *free*.

Given a finite set S, a basic probability assignment (bpa) over S is a function $p : \mathcal{P}(S) \to [0,1]$ with $p(\emptyset) = 0$ and $\sum_{S' \subseteq S} p(S') = 1$. The set of all basic probability assignments over S is denoted by $\mathit{BPA}(S)$. With this, a basic sensor function can be generalised into its probabilistic variant as follows.

Definition 3 (Uncertain Sensor). *Let $P = (\mathcal{E}, \varsigma, \iota, \mathcal{C})$ be a perception. A function*

$$\tilde{\varsigma} : \mathcal{E} \to \mathit{BPA}(\mathcal{C})$$

that maps each environment situation to a bpa over $\mathcal{C}$ is called an uncertain sensor *over P. The set of all uncertain sensors over P is denoted by $\mathit{US}(P)$.* $\diamondsuit$

By using a bpa, an uncertain sensor can map dedicated probability masses in particular to non-singleton contexts, thereby expressing a certain amount of inconclusive observations and even uncommitted belief (assignments to $\mathcal{C}$).

3.1 Sensor Quality

An uncertain sensor expresses to which amount it yields different outcomes than the sensing function as given by the perception. For the running example, an uncertain sensor with

$$\tilde{\varsigma}(\{c\})(\{o\}) = 0.7 \qquad \tilde{\varsigma}(\{c\})(\{f\}) = 0.1 \qquad \tilde{\varsigma}(\{c\})(\{o, f\}) = 0.2$$

states that with a probability of 70% the sensor coincides with ς but with 10% it yields $\{f\}$ and with 20% $\{o, f\}$. By taking the interpretation function into account we can classify this amount of incorrect sensing into unsafety and imprecision: As $\{c\} \not\subseteq \iota(\{f\})$, $\tilde{\varsigma}$ is 10% unsafe, and as $\{c\} \subsetneq \iota(\{o, f\})$ it is 20% imprecise. We generalise this observation in the following definitions.

Definition 4 (Environment Classification). *Let $P = (\mathcal{E}, \varsigma, \iota, \mathcal{C})$ be a perception and $e \in \mathcal{E}$ an environment situation. Then*

$$\begin{aligned}
\mathit{Cor}_P(e) &= \{E \in \mathcal{P}(\mathcal{E}) \mid \iota(\varsigma(\{e\})) = \iota(\varsigma(E))\} \\
\mathit{Imp}_P(e) &= \{E \in \mathcal{P}(\mathcal{E}) \mid \iota(\varsigma(\{e\})) \subsetneq \iota(\varsigma(E))\} \\
\mathit{Uns}_P(e) &= \mathcal{P}(\mathcal{E}) \setminus \big(\mathit{Cor}_P(e) \cup \mathit{Imp}_P(e)\big)
\end{aligned}$$

denote the correct, imprecise and unsafe environments for e, respectively. $\diamondsuit$

To motivate this definition, we introduce a more elaborated example which is inspired by [17]. The task is to measure the distance to an object within a radius of zero to nine units. The sensor has a rather low resolution and can only distinguish between the distances *(c)lose*, *(n)ear* and *(f)ar*. More specifically, setting $\mathcal{E} = \{0, \ldots, 9\}$ and $\mathcal{C} = \{c, n, f\}$, the basic sensor function is

$$\varsigma_0(e) = \begin{cases} \{c\} & \textit{if } 0 \leq e \leq 2 \\ \{n\} & \textit{if } 3 \leq e \leq 5 \\ \{n, f\} & \textit{if } e = 6 \\ \{f\} & \textit{if } 7 \leq e \leq 9 \end{cases}$$

and ς and ι are obtained by proposition 2. For example, we obtain $\varsigma(\{2, 3\}) = \{c, n\}$, and $\iota(\{n\}) = \{3, 4, 5\}$, and $\iota(\{n, f\}) = \{3, \ldots, 9\}$. We will refer to this perception by the name D in the following.

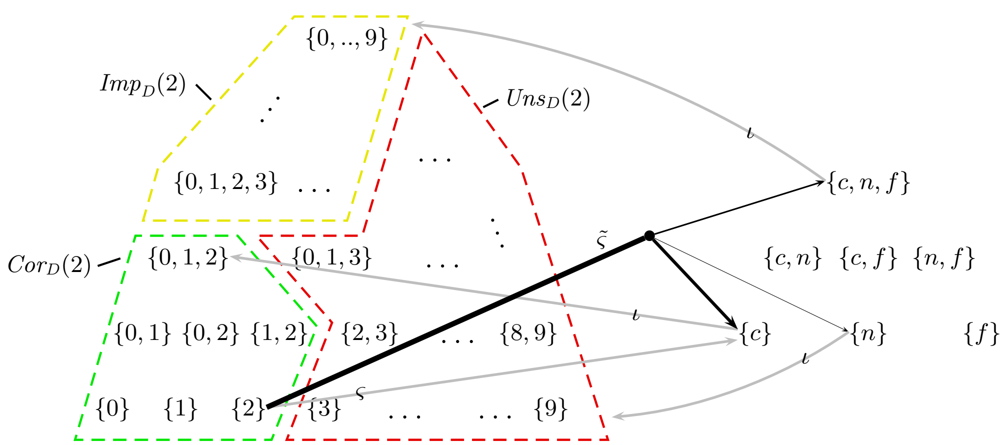


Fig. 2: Illustration of Def. 4 and Def. 5 for $e = 2$ with $\iota(\varsigma(\{2\})) = \{0, 1, 2\}$

Figure 2 illustrates the concepts of Def. 4. For the environment situation 2, the sensing function derives the context $\{c\}$ which is interpreted as $\{0, 1, 2\} =: E$. This induces a partitioning into correct environments $Cor_D(2)$ (subsets of E and E itself), imprecise environments $Imp_D(2)$ (proper supersets of E), and unsafe environments $Uns_D(2)$ (neither subsets nor supersets of E). Note that in particular $\{0\}$ is considered to be a correct environment for $\{2\}$, basically as the perception cannot distinguish them anyway.

We obtain a measure of unsafety and imprecision by summing the probability masses of those contexts which are interpreted to environments of the corresponding partition. This is illustrated in Fig. 2 by the $\tilde{\varsigma}$-labelled arrow which branches the sensing into different contexts with certain probabilities.

Definition 5 (Environment Quality). *Let $P = (\mathcal{E}, \varsigma, \iota, \mathcal{C})$ be a perception and $\tilde{\varsigma} \in US(P)$ an uncertain sensor. The environment quality of $\tilde{\varsigma}$ for an environment situation $e \in \mathcal{E}$ is determined by*

$$Uns_{\tilde{\varsigma}}(e) := \sum_{O \in \mathcal{P}(\mathcal{C}),\, \iota(O) \in Uns_P(e)} \tilde{\varsigma}(e)(O)$$

$$Imp_{\tilde{\varsigma}}(e) := \sum_{O \in \mathcal{P}(\mathcal{C}),\, \iota(O) \in Imp_p(e)} \tilde{\varsigma}(e)(O)$$

which quantify the amount of unsafety and imprecision, respectively.

The average values over all environment situations are denoted by $Uns_{\tilde{\varsigma}}(\mathcal{E}) := 1/|\mathcal{E}| \cdot \sum_{e \in \mathcal{E}} Uns_{\tilde{\varsigma}}(e)$ and $Imp_{\tilde{\varsigma}}(\mathcal{E}) := 1/|\mathcal{E}| \cdot \sum_{e \in \mathcal{E}} Imp_{\tilde{\varsigma}}(e)$. ♢

We can obtain a similar notion of quality from the *perspective of a context*. This is of particular relevance for practical applications as the system only has information about the context as given by its sensor implementation, and does not know the real environment. A notion of context quality then allows for an estimation how reliable this context information with respect to the actual environment situation is. Formally, the quantity can be computed as the conditional probability of unsafe and imprecise sensings with respect to the probability for all possible environment situations that map to a certain context.

Definition 6 (Context Quality). *Let $P = (\mathcal{E}, \varsigma, \iota, \mathcal{C})$ be a perception and $\tilde{\varsigma} \in US(P)$ an uncertain sensor. The context quality of $\tilde{\varsigma}$ for a context $O \in \mathcal{P}(\mathcal{C})$ is determined by*

$$Uns_{\tilde{\varsigma}}(O) := \frac{\sum_{e \in \mathcal{E},\, \iota(O) \in Uns_P(e)} \tilde{\varsigma}(e)(O)}{\sum_{e \in \mathcal{E}} \tilde{\varsigma}(e)(O)}$$

$$Imp_{\tilde{\varsigma}}(O) := \frac{\sum_{e \in \mathcal{E},\, \iota(O) \in Imp_P(e)} \tilde{\varsigma}(e)(O)}{\sum_{e \in \mathcal{E}} \tilde{\varsigma}(e)(O)}$$

which quantify the amount of unsafety and imprecision, respectively. ♢

For example, we consider a sensor $\tilde{\varsigma}_1 \in US(D)$ that is well suited to detect close objects, but tends to make errors for larger distances. The probability function of this sensor is given in Table 1. The last two columns given the amount of unsafety and imprecision for each environment situation for $\tilde{\varsigma}_1$ with respect to D according to Def. 5, and the last two rows gives the corresponding values for the contexts according to Def. 6.

In the example, sensing the environment 7 yields with a probability of 0.3 an unsafe context. This is as $\iota(\varsigma(\{7\}))$ is $\{7, 8, 9\}$ and hence both sets $i(\{c\}) = \{0, 1, 2\}$ and $i(\{n\}) = \{3, 4, 5\}$ are in $Uns_D(7)$. Thus $Uns_{\tilde{\varsigma}_1}(7) = \tilde{\varsigma}(7)(\{c\}) + \tilde{\varsigma}(7)(\{n\}) = 0.1 + 0.2 = 0.3$. Analogously, we obtain a probability of 0.2 for being imprecise as both $\iota(\{n, f\})$ and $\iota(\{c, n, f\})$ are elements of $Imp_D(7)$. The average amount of unsafety and imprecision of $\tilde{\varsigma}_1$ is 0.09 and 0.23, respectively.

As an example for context quality, we consider the context $\{c, n\} =: O$. The sum of probabilities over all environments is $\sum_{e \in \mathcal{E}} \tilde{\varsigma}(e)(O) = 0.55$. We

e O	$\{c\}$	$\{n\}$	$\{f\}$	$\{c,n\}$	$\{n,f\}$	$\{c,f\}$	$\{c,n,f\}$	$Uns_{\tilde{\varsigma}_1}(e)$	$Imp_{\tilde{\varsigma}_1}(e)$
0	1.0							0	0
1	0.95			0.05				0	0.05
2	0.9			0.1				0	0.1
3		0.8		0.2				0	0.2
4		0.9		0.1				0	0.1
5		0.65	0.05		0.3			0.05	0.3
6		0.15	0.15		0.6		0.1	0.3	0.1
7	0.1	0.2	0.5		0.15		0.05	0.3	0.2
8		0.1	0.4		0.2		0.3	0.1	0.5
9			0.3	0.1			0.7	0.1	0.7
$Uns_{\tilde{\varsigma}_1}(O)$	0.03	0.11	0.14	0.18	0	-	0	$Uns_{\tilde{\varsigma}_1}(\mathcal{E})$	$Imp_{\tilde{\varsigma}_1}(\mathcal{E})$
$Imp_{\tilde{\varsigma}_1}(O)$	0	0	0	0.82	0.35	-	1	0.09	0.23

Table 1: Uncertain sensor $\tilde{\varsigma}_1$ together with its quality masses. The gray cells visualise the sensing function given by the perception D.

have $\iota(O) = \{0, \ldots, 5\} \in Imp_D(e)$ for $e \in \{1, 2, 3, 4\}$. This yields $Imp_{\tilde{\varsigma}_1}(O) = (0.05 + 0.1 + 0.2 + 0.1)/0.55 = 0.82$. Analogously, as $\iota(O) \in Uns_D(9)$ we obtain $Uns_{\tilde{\varsigma}_1}(O) = 0.1/0.55 = 0.18$. Note that we have $Uns_{\tilde{\varsigma}_1}(O) + Imp_{\tilde{\varsigma}_1}(O) = 1$ due to the fact that $\{c, n\}$ is not in the image of ς_0, and hence there is no environment which *correctly* maps to it.

All in all, the quality values reflect the intuition that the sensor is rather safe when delivering a (c)lose distance (0.03), but becomes unsafer for distances (n)ear and (f)ar (0.11 and 0.14). Additionally, from $Imp_{\tilde{\varsigma}_1}(\mathcal{E}) = 0.23$ we see that the sensor gives an imprecise reading in almost every forth case (assuming that all environment situations occur with an equal probability).

3.2 Properties of the Quality Mass

We present a number of properties regarding our quality mass. The first observation is a direct consequence of the definition of an uncertain sensor $\tilde{\varsigma}$ based on basic probability assignments, and of the fact that the sets of correct, imprecise and unsafe environment indeed form a partition of all possible environments.

Proposition 4 (Quality Mass). *Let $P = (\mathcal{E}, \varsigma, \iota, \mathcal{C})$ be a perception and $\tilde{\varsigma} \in US(P)$ an uncertain sensor. Then both the environment and the context quality lies in $[0, 1]$, that is,*

(i) $0 \leq Uns_{\tilde{\varsigma}}(e) + Imp_{\tilde{\varsigma}}(e) \leq 1$
(ii) $0 \leq Uns_{\tilde{\varsigma}}(O) + Imp_{\tilde{\varsigma}}(O) \leq 1$

for any environment $e \in \mathcal{E}$ and context $O \in \mathcal{P}(\mathcal{C})$. ♢

For the context quality, we have certain probabilities fixed to zero for singleton contexts and the universal context. When working with *safe* perceptions, the context $\mathcal{C}$ cannot be unsafe as is represents all possible environments. Similarly, the imprecision of singleton contexts is zero as, due to the monotonicity, there is no smaller context which is potentially more precise.

Proposition 5 (Definite Context Quality). *Let $P = (\mathcal{E}, \varsigma, \iota, \mathcal{C})$ be a safe perception and $\tilde{\varsigma} \in US(P)$ an uncertain sensor. Then $Uns_{\tilde{\varsigma}}(\mathcal{C}) = 0$ and $Imp_{\tilde{\varsigma}}(\{o\}) = 0$ for any $o \in \mathcal{C}$.* ♢

For an *exact* perception, we observe a relationship of our quality mass to the classical notion of belief and plausibility in the Dempster-Shafer framework [5,14]. Given a finite set Ω and a bpa $m \in BPA(\Omega)$, the two functions

$$Bel_m(X) := \sum_{X' \subseteq X} m(X') \qquad\qquad Pl_m(X) := \sum_{X' \cap X \neq \emptyset} m(X')$$

define the belief and the plausibility regarding a subset $X \subseteq \Omega$, respectively. Assuming an exact perception $P = (\mathcal{E}, \varsigma, \iota, \mathcal{C})$ and an uncertain sensor $\tilde{\varsigma} \in US(P)$ with $\tilde{\varsigma}(e) = m$, we have

$$Pl_m(\{e\}) = 1 - Uns_{\tilde{\varsigma}}(e) \tag{1}$$

and

$$Bel_m(\{e\}) = 1 - (Uns_{\tilde{\varsigma}}(e) + Imp_{\tilde{\varsigma}}(e)) \tag{2}$$

for any environment situation e. In particular, the difference between belief and plausibility corresponds to our notion of imprecision.

The proof for this relationship exploits that $\iota(\varsigma(\{e\}))$ equals $\{e\}$ in an exact perception. Then the belief measure exactly corresponds to our notion of correct environments according to Def. 4, which entails equation 2. Moreover, as

$$Cor_P(e) \cup Imp_P(e) = \{E' \in \mathcal{E} \mid E' \cap \{e\} \neq \emptyset\}$$

we have $Bel_m(\{e\}) + Imp_{\tilde{\varsigma}}(e) = Pl_m(\{e\})$, which entails equation 1.

3.3 Sensor Fusion

The relation to the Dempster-Shafer framework also allows us to evaluate the effects of sensor fusion. To this end, we choose another uncertain sensor $\tilde{\varsigma}_2$ which gives good results as long as the obstacle is near or far, but is very imprecise on smaller distances. The bpa are given in Table 2. We see that the sensor does not give any unsafe output, but produces imprecise results on about every second reading. Combining the information of both sensors $\tilde{\varsigma}_1$ and $\tilde{\varsigma}_2$ will hopefully yield a sensor that gives safe and precise results for the complete set of environments.

Dempster's rule of combination [5] merges two bpa $m_1, m_2 \in BPA(S)$ by setting $(m_1 \oplus m_2)(\emptyset) = \emptyset$ and

$$(m_1 \oplus m_2)(X) = \frac{\sum_{A \cap B = X} m_1(A) \cdot m_2(B)}{1 - \sum_{A \cap B = \emptyset} m_1(A) \cdot m_2(B)}$$

e \ O	$\{c\}$	$\{n\}$	$\{f\}$	$\{c,n\}$	$\{n,f\}$	$\{c,f\}$	$\{c,n,f\}$	$Uns_{\tilde{\varsigma}_2}(e)$	$Imp_{\tilde{\varsigma}_2}(e)$
0							1.0	0	1.0
1							1.0	0	1.0
2	0.1			0.2			0.7	0	0.9
3		0.2		0.2			0.6	0	0.8
4		0.5		0.2			0.3	0	0.5
5		0.4			0.5		0.1	0	0.6
6					0.6		0.4	0	0.4
7			0.7		0.1		0.2	0	0.3
8			0.9				0.1	0	0.1
9			1.0					0	0
$Uns_{\tilde{\varsigma}_2}(O)$	0	0	0	0	0	-	0	$Uns_{\tilde{\varsigma}_2}(\mathcal{E})$	$Imp_{\tilde{\varsigma}_2}(\mathcal{E})$
$Imp_{\tilde{\varsigma}_2}(O)$	0	0	0	1	0.5	-	1	0	0.56

Table 2: Uncertain sensor $\tilde{\varsigma}_2$ together with its quality masses.

for $X \in \mathcal{P}(S) \setminus \{\emptyset\}$. Given a set of hypotheses X, this rule combines individual probability masses whenever there is an agreement (in the sense of intersection) of both bpa regarding the set X. Conflicting evidence where the intersection is empty is distributed over all values by a normalisation factor.

e \ O	$\{c\}$	$\{n\}$	$\{f\}$	$\{c,n\}$	$\{n,f\}$	$\{c,f\}$	$\{c,n,f\}$	$Uns_{\tilde{\varsigma}_3}(e)$	$Imp_{\tilde{\varsigma}_3}(e)$
0	1.0							0	0
1	0.95			0.05				0	0.05
2	0.91			0.09				0	0.09
3		0.84		0.16				0	0.16
4		0.95		0.05				0	0.05
5		0.75	0.03		0.18			0.03	0.18
6		0.15	0.15		0.66		0.04	0.3	0.04
7	0.03	0.08	0.82		0.06		0.01	0.11	0.07
8		0.01	0.94		0.02		0.03	0.01	0.05
9			1.0					0	0
$Uns_{\tilde{\varsigma}_3}(O)$	0.01	0.09	0.06	0	0	-	0	$Uns_{\tilde{\varsigma}_3}(\mathcal{E})$	$Imp_{\tilde{\varsigma}_3}(\mathcal{E})$
$Imp_{\tilde{\varsigma}_3}(O)$	0	0	0	1	0.28	-	1	0.05	0.07

Table 3: Fusion of $\tilde{\varsigma}_3 = \tilde{\varsigma}_1 \oplus \tilde{\varsigma}_2$ together with its quality masses.

Under the assumption that both sensors are independent sources, we can use this rule to combine the individual bpa for each environment situation. The result for the example is given in Table 3. We see that the safe reading of $\tilde{\varsigma}_2$ for (in particular) near and large distances is able to reduce the amount of unsafety for the corresponding contexts. Also, the overall amount of imprecision is reduced to 0.07. Note that the unsafety of environment situation 6 can not be reduced by the rule of combination as given above. This is however of no particular surprise as $\varsigma_0(6) = \{n, f\}$ violates the Dempster-Shafer assumption of working with *exclusive* hypotheses. A possible solution would either be to focus on basic sensor functions that map to singleton contexts only, or to integrate the modified theory of Dempster-Shafer (DSmt, see [6]) where the assumption of exclusive hypotheses is removed. Technically, this requires to go from powersets to hyper-powersets in order to represent disjunctive versus conjunctive hypotheses.

4 Related Work

Context-awareness [13,1], context reasoning [16,1] and Quality of Context [9,10] have been studied as general concepts and methods for pervasive computing applications. Without a good "understanding" of their working context, such devices cannot realise the vision of smart and ubiquitous computing. As soon as smart devices start to act autonomously in a heterogeneous environment, a new "quality of quality" has to be considered as wrong context information can lead to severe hazards. This problem has in particular been recognised for autonomous vehicles [18,12,8], however no widely accepted formal framework for a thorough investigation of safety-critical context information exists.

Lattice theory is already heavily used in context modelling and reasoning. For example, formal concept analysis [7] derives a structured view of large amounts of raw context data by means of ordered structures (concept lattices). This is used both as a machine learning technique and for data analysis. Situations are considered as a semantic interpretation of context information, and lattices of situations [19] capture the dependence between situations in a structured way. Run-time learning on the basis of lattices is for example done in [20].

In the framework of Abstract Interpretation [3,11], Galois connections are used for constructing safe approximations of program semantics. There, the credo is to "analyse the abstract and conclude to the concrete behaviour" of (in particular safety-critical) software by means of formal methods. The same principle applies to our setting as we "decide on the context and act in the environment". In fact, this "analogy" was the starting point for our investigation.

5 Conclusion

Based on the relation between the real environment and the derived context, we presented a hierarchy of quality notions which are inspired by the theory of Galois connections. Using these notions as a basic qualitative assessment, we developed an evaluation method for the intrinsic uncertainty of real-world sensor applications. This method derives a measure of safety and precision, which are the two relevant parameters for safety and liveness aspects [15] of systems. We showed that our approach generalises the classical belief/plausibility measures to non-exact perceptions, i.e. perceptions with an inherent loss of information.

To keep this paper focussed on the main ideas, we used a rather simple context and environment representation. However, our approach is extendable to any lattice-structured representation, and in future work we will transfer the ideas to more sophisticated context modelling approaches [1]. The integration of techniques like DSmt [6] or conflict metrics [2] is possible and desirable. Also, an evaluation with realistic sensor data will investigate the applicability of our high-level formalisation to a real-world implementation of a sensor perception.

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